## Lesson 1, The Trigonometry of Retirement Income

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It is generally believed that stock markets, interest rates and investment returns move in periodic cycles. Unfortunately, these cycles are swamped with noise and only become evident with the benefit of hindsight. The current academic consensus is that they are extremely difficult to predict or measure in advance. Nevertheless, these cycles — once they do materialize — can have a profound impact on the sustainability of retirement income.

If you retire and start to withdraw money from a diversified investment portfolio just as the economy moves into a bear-market cycle your portfolio's longevity can be at risk. Your nest egg will not last as long as it would under an equivalent spending plan which started during a bull-market cycle. This observation is often labeled by the term "sequence-of-returns" risk and is used by many in the insurance industry to explain the importance of downside protection during the so-called "retirement risk zone."

I, too, have elaborated elsewhere on the role of guarantees early-on in retirement using analogies of spinning triangles, roulette wheels and Monte Carlo simulations. In this brief article I would like to take a slightly different approach to the issue — one that hopefully balances realism and accessibility.

My plan here is to illustrate exactly how a bull or bear market cycle can impact the sustainability of your portfolio by appealing to ideas from basic high school trigonometry, a.k.a. the mathematics of sine and cosine waves. And, although your skills might be rusty after these many years — and I'm not even sure they teach this anymore in high school — the main story should be accessible to all.

First, allow me to review the basic arithmetic of generating income, sustainability and ruin. Assume you start retirement with a nest egg of exactly $100 and you allocate this to an investment fund that earns a real (i.e., after inflation) 5 percent per annum during every year of your retirement. For simplicity, I take this 5 percent to be an annual percentage rate (APR) that compounds continuously over time. Remember, this implies that if inflation is 3 percent per annum, then your nominal return is (approximately) 5% + 3% = 8%. If inflation is 4 percent, then your nominal return is (approximately) 9 percent.

Now let's spend some money. If you withdraw $6 (also inflation-adjusted) per year from this portfolio the nest egg will be exhausted — and your retirement ruined — in exactly 35.8 years. In contrast, if you withdraw $7 (inflation-adjusted) per year the funds will last for 25.1 years. At $8 of spending you will be ruined in 19.6 years. Remember, all of this assumes your portfolio earns the same consistent inflation-adjusted 5 percent APR for ever.

At this point I need you to suspend your disbelief and imagine a perfectly cyclical (sine wave) financial market; and I mean perfect with no randomness, noise or real-world

uncertainty. I'm going to describe two symmetrically opposed scenarios. Look at Figure No. 1 for a picture that's worth the next 200 words.

In that figure, the blue line represents how the portfolio starts out earning an APR of 5 percent on the first day of retirement. To be exact, this is 2 basis points during the first day of retirement, which is a 5 percent APR divided by 250 trading days. The market then moves into a bull-market cycle so that your annualized returns slowly increase until it peaks at 20 percent per annum (8 basis point per day) in approximately 19 months. In the language of sine waves, the market peaks after approximately π/2 years. Remember the Greek letter π is equal to approximately 3.14 (years), which is 37.68 months, so π/2 is just shy of 19 months.

Then after hitting this peak financial markets start to decline so that approximately 19 additional months later the market is back to earning an APR of 5 percent. It has gone from 5 percent up to 20 percent and then down to 5 percent over approximately 3.14 years.

Bear with me here. Imagine that markets then continue to decline for another 19 months and your portfolio's investment return hits -10 percent annualized (which is -4 basis points per day). Please see the blue line in Figure #1 for an illustration of the evolution of this entire cycle from start to finish over 2π (= 6.28) years. This sine wave I have constructed exhibits the amplitude (volatility) of plus or minus 15 percent, and ranges in value from negative 10 percent to positive 20 percent.

As a perfectly symmetric alternative, consider the scenario in which you retire and start to withdraw funds while the market is earning the same 5 percent per annum, but it immediately moves into a bear-market cycle so that 19 months into retirement you are earning -10 percent (i.e. losing money) per annum, and 19 months later you are back to 5 percent per annum and 19 months after that you are earning 20 percent, etc.

Once again, the two diametrically opposed paths are illustrated in Figure No. 1. All paths earn an average APR of 5 percent over the long run.

So far I have not mentioned withdrawal rates or values from the portfolio. I will get to that in a moment. For now it is important to notice that if you only invested $1 at the start of a cycle (up or down, bullish or bearish) then at the end of a market cycle of 2π (= 6.28) years you would have a compound annual return of 5 percent and the same amount of money, $1.37, in all cases.

I have reached the first of my two main points. When you are buying and holding it doesn't matter what path the market takes as long as you get the 5 percent annualized return! Whether you move up first and then down, or down first and then up, if your compound return is 5 percent you will know exactly where you end up. But the situation is very different when you withdraw money.

Next let's look at what happens when you start retirement with $100 and withdraw $6, $7 or $8 per year on a continuous basis — under fluctuating as opposed to fixed returns. This all scales up to $100,000 with $6,000 withdrawals or a million dollars with $60,000 withdrawals.

I trust you would agree that on any given day, week or month, you don't really know in which direction the market will move over the next few years. It might go north or it might go south. The relevant question is how long your portfolio will last based on the possible starting market cycles. It should be intuitive that your retirement will be worse off if markets go south versus north. But how bad will it be? That's my second point.

Figure No. 2 tells the story and Table No. 1 summarizes the main results. For example, if you are withdrawing $7 per year and you are invested in a relatively volatile asset class that fluctuates between +20 percent and -10 percent, the portfolio can last anywhere between 34.9 years to 18.9 years depending on where — in the cycle — you start retirement. Figure No. 2 is a graphical illustration of the possible range of outcomes.

As you can see from the adjacent table, if you invest in even more volatile (higher amplitude) asset classes that range from +25 percent to -15 percent per year, for example, the dispersion of outcomes is an even wider 17.1 years to 39.6 years.

Another important insight from the table of numbers is the impact of the spending rate itself. Notice that when you are spending $8 per year the gap between the "good" sequence and the "bad" sequence of returns is only 21.3 – 18 = 3.3 years in the first row (low risk) of the table. This same gap is 41.7 – 31.4 = 10.3 years when spending is reduced to $6 per year. At first glance this might seem odd. Why is the sequence-of-returns effect more powerful at lower spending rates? But of course, this is a relative effect. If you spend more (i.e., $8) you will be worse off in terms of sustainability regardless of the sequence. That said , the gap between worst- and best-case scenarios increases the less you spend. This impact becomes more pronounced at greater volatility levels. *Starving yourself — i.e., withdrawing less — might push back your date with retirement ruin, but it won't immunize you from a bad initial sequence-of-returns.*

In sum, no economist can tell you with any degree of accuracy whether the next few years will be sine-up or sine-down. It only becomes evident over time. The practical implications of all of this? When it comes time to withdraw, don't take sequence-of-returns chances and make sure to protect your nest egg during the fragile first decade of retirement. We will explore this in future lessons. Next month I will examine how income taxes throw another wrench into the retirement dynamics.

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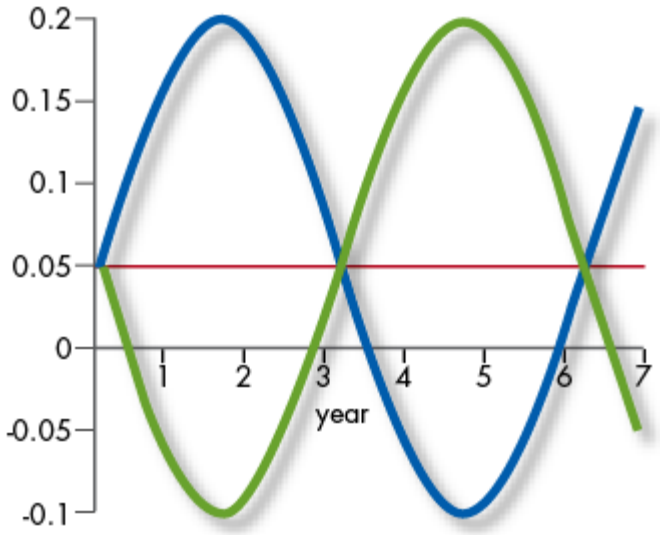
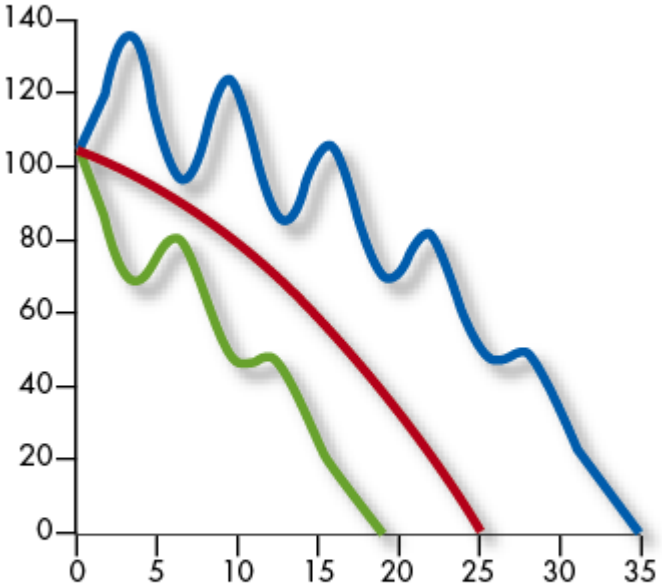
**Figure 1. Markets will cycle over time even if you earn 5 percent on average.**




Note: The flat (red) line denotes a portfolio that earns 5 percent every year during retirement. In contrast, if you start retirement in a bull-market cycle your portfolio will experience above average returns during the first three years (blue line). But, if you enter a bear-market cycle you will earn below average returns during the first three years (green line). In either case you will earn the same 5 percent average over the long run. In the above picture, the range of returns is (-10 percent, +20 percent.)

**Figure 2. Retirement in a bear versus bull cycle. Spending $7 per $100 of initial nest egg.**




Note: The smooth middle (red) curve denotes your portfolio's retirement path assuming it earns 5 percent per year, every year. The upper (blue) curve assumes you enter retirement in a bull-market cycle and the lower (green) curve assumes you start in a bear-market cycle. The sustainability gap between the two extreme scenarios is approximately 16 years.

**Table 1: How Long Does the Money Last in Retirement?**
**Assuming Your Portfolio Cycles around an Average 5 percent Return per Year**

| SEQUENCE OF RETURNS | $100 Initial Nest Egg and Spending... | | | |
|---|---|---|---|---|
| | Market Cycle | $6 per year | $7 per year | $8 per year |
| N.A. | Flat 5 percent Market | 35.8 yrs. | 25.1 yrs. | 19.6 yrs. |
| +5,+10,+5,0 | Retire into Bull | 41.7 | 27.7 | 21.3 |
| +5,0,+5,+10 | Retire into Bear | 31.4 | 22.7 | 18.0 |
| +5,+15,+5,-5 | Retire into Bull | 50.1 | 31.0 | 23.4 |
| +5,-5,+5,+15 | Retire into Bear | 27.9 | 20.8 | 16.5 |
| +5,+20,+5,-10 | Retire into Bull | 65.6 | 34.9 | 25.5 |
| +5,-10,+5,+20 | Retire into Bear | 24.9 | 18.9 | 15.4 |
| +5,+20,+5,-10 | Retire into Bull | Infinity | 39.6 | 28.1 |
| +5,-10,+5,+20 | Retire into Bear | 22.4 | 17.1 | 14.3 |

Note: The first row displays the time (year) at which you exhaust the retirement portfolio, subjected to spending rates of $6 or $7 or $8 per initial $100 nest egg and assuming you continuously earn 5 percent (after inflation). The remaining rows display the time (year) of ruin assuming you earn 5 percent on average, but cycle between various extremes.